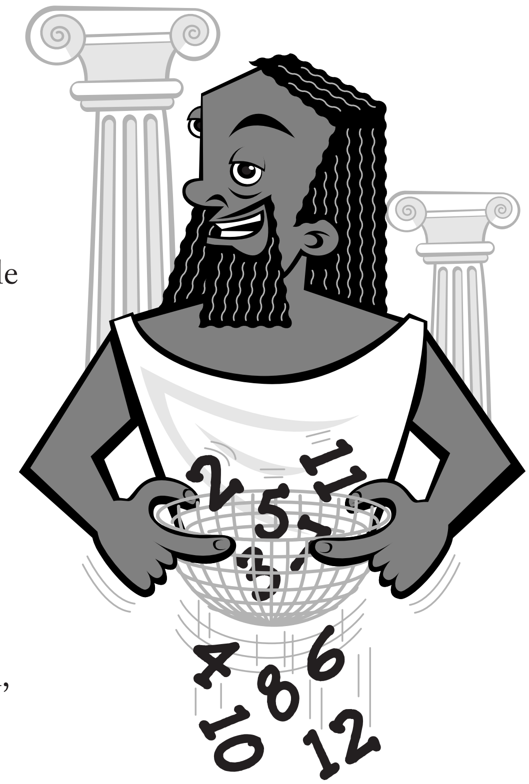
ACTIVITY 6

Eratosthenes' Sieve



Many middle schoolers struggle with the multiplication table and problems that require this basic knowledge. This activity will help those who have difficulty with multiplication.

This activity also introduces prime numbers. A prime number is a number that can be divided evenly only by 1 and itself. "Divided evenly" means that the result is a whole number, with no remainder. Prime numbers and their properties were first studied extensively by ancient Greek mathematicians, including Eratosthenes (pronounced AIR-a-TOSS-the-knees).

Preparation and Materials

Each member of your group will need:

- a pencil
- 4 different-colored pens or markers (yellow, red, green, and blue)
- a copy of *Hundreds Chart*

This activity works best if you lead your group through the steps, using *Eratosthenes' Sieve* as your guide. If you want people to work independently, each will need a copy of *Eratosthenes' Sieve*.

Using This Activity

By completing *Eratosthenes' Sieve*, people will acquire a tool to help them play *Hopping Hundred* (page 55) and *Tic-Tac-Toe Times* (page 61), games that require knowledge of the multiplication table. You may want to do this activity right before playing one or both of those games.

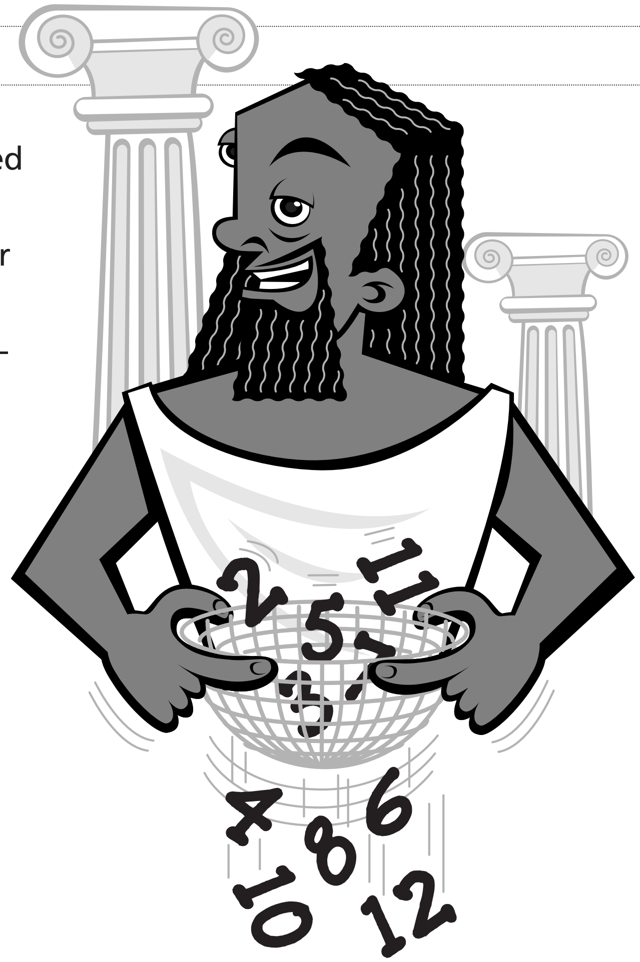
Tips for how to use *Eratosthenes' Sieve* start on page 53.

Planning chart

Finding the prime numbers by using *Eratosthenes' Sieve*	20 minutes

Explorer's Notebook

Eratosthenes' Sieve



If you've ever made cookies, you may have used a sieve to sift flour. A *sieve* is a device with a bottom that's full of tiny holes. It lets fine flour particles through but retains the lumpy bits.

Eratosthenes (pronounced AIR-a-TOSS-the-knees) was an ancient Greek athlete, astronomer, and mathematician. His sieve doesn't sort out lumps of flour. It sorts out prime numbers. A *prime number* is a number that can be divided evenly only by 1 and itself. "Divided evenly" means that the result is a whole number, with no remainder.

What Do I Need?

- a pencil
- a *Hundreds Chart* (page 52)
- 4 different-colored pens or markers (yellow, red, green, and blue)

What Do I Do?

When you mark your *Hundreds Chart* according to the steps that follow, you'll reveal all the prime numbers in the chart. At the same time, you'll discover patterns that will help you remember the multiplication table.

Step 1 Using your pencil, draw an *X* through the number 1 on your *Hundreds Chart*. The number 1 is a special number. It's not prime, because a prime number has to be evenly divisible by two numbers: 1 *and* itself. The number

1 is divisible by only one number: 1. So, it's not prime. (That may seem picky, but mathematicians are like that sometimes.)

Step 2 Look at the number 2. The number 2 is prime because only two numbers can divide into it evenly: 1 and 2. Circle the 2 with your pencil.

Step 3 Using your yellow marker, draw a vertical line through the center of the box the number 2 is in.

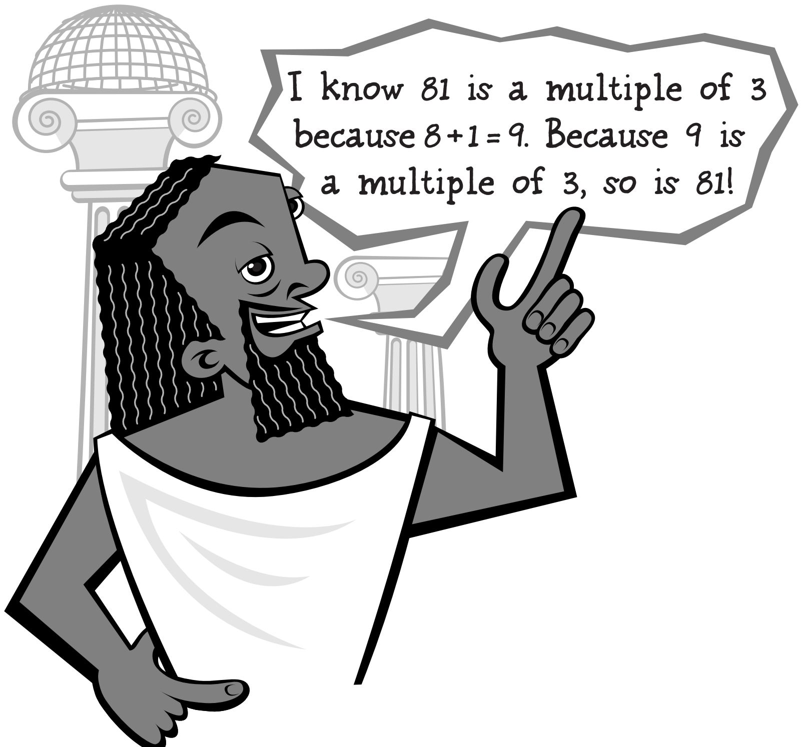
Step 4 Find all the other multiples of 2 in the chart, and draw the same yellow vertical line through each of them. Is 3 a multiple of 2? How about 4? The number 4 is a multiple of 2 because it equals 2×2.

To find the multiples of 2, look for the *even* numbers. Be sure to draw a yellow vertical line through every even number between 2 and 100. Do you see a pattern in the multiples of 2?

Step 5 At the bottom of your chart, you'll find a place to create a key to help you remember what the different colors indicate. Draw a yellow vertical line through the box labeled "Multiples of 2." Any number that has a yellow vertical line through it is a multiple of 2.

Step 6 Look at the next number in the top row: 3. Is 3 a prime number? It is, because it can be divided evenly only by 1 and 3. Circle the 3 with your pencil.

Step 7 Using your red marker, draw a diagonal line across the box containing the number 3. Make the diagonal from the bottom left corner to the top right corner.



Step 8 Using the red marker, draw the same diagonal through all the multiples of 3 in the chart. The number 6 is a multiple of 3; it is 3 multiplied by 2. Find all the multiples of 3 between 3 and 100. After you've marked several multiples of 3, you may be able to find a pattern to help you locate the others.

There is also a trick for finding the multiples of 3. Add up the digits, and if that sum is a multiple of 3, then the number itself is a multiple of 3.

Step 9 In the key at the bottom of your chart, draw a red diagonal line through the box labeled "Multiples of 3."

Notice that some boxes in your chart have both a yellow vertical line and a red diagonal line. These numbers are multiples of both 2 and 3. If a number is a multiple of two numbers, it's also a multiple of those two numbers multiplied together! So, these boxes show you the multiples of 2×3, or 6. Mark this on your key, too, by drawing both a yellow vertical line and a red diagonal line in the box labeled "Multiples of 6."

Step 10 Continuing along the top row of the chart, notice that 4 is already marked with a yellow vertical line, reminding you that it's a multiple of 2, so it can't be a prime number. The next number, 5, is prime, so circle it with your pencil.

Using your green marker, draw a vertical line through the 5. Draw this green line a little left of center so that you don't cover up any of the yellow lines you already drew in the multiples of 2.

Step 11 Find all the multiples of 5 in the chart, and draw the same green line through each of them. The trick to finding multiples of 5 is to look for numbers that end in 5 or zero. Look for a pattern that will help you find all the multiples of 5. In your key, draw a green line through the box labeled "Multiples of 5."

Step 12 Looking at your key and your chart, can you figure out what set of colored lines shows you the multiples of 10? (Here's a hint: $2 \times 5 = 10$.) Now look for multiples of 15 (which is 3×5) and 30 (which is $3 \times 5 \times 2$). Add these to your key.

Step 13 The number 6 has already been marked. The number 7 is the next prime. Circle 7 with your pencil. Using your blue marker, draw a diagonal line across 7. This time, draw the diagonal from the top left corner to the bottom right corner so that it doesn't cover up the diagonal you drew for multiples of 3.

Step 14 Find all the multiples of 7 in the chart, and draw a blue diagonal line through each of them. There is no trick for finding multiples of 7, but these numbers do form a pattern on your chart. Look for the pattern.

Step 15 In your key, draw a diagonal blue line through the box labeled “Multiples of 7.” If you want to, you can add other boxes to your key. Can you figure out what set of colored lines shows you multiples of 14? Or 21? Or 35?

Step 16 Continuing along the chart, notice that 8, 9, and 10 are already marked. There are no prime numbers left in the top row. All the numbers that haven't been marked yet are the remaining primes between 1 and 100. Circle all the remaining numbers.

Step 17 How many prime numbers are there between 1 and 100? You should get the same answer as everyone else in your group.

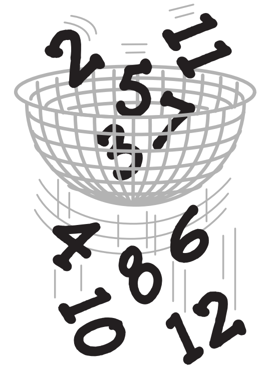
Explorer's Notebook

Explorer's Name: Date:

Hundreds Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- ☐ Multiples of 2
- ☐ Multiples of 3
- ☐ Multiples of 6
- ☐ Multiples of 5
- ☐ Multiples of 10
- ☐ Multiples of 15
- ☐ Multiples of 30
- ☐ Multiples of 7



Leading Your Group Through *Eratosthenes' Sieve*

By taking a different approach, this activity may help people who have never managed to memorize the multiplication table. Mastery of the multiplication table is important for middle schoolers because this knowledge is essential to working effectively with fractions and doing algebra.

About Eratosthenes

Eratosthenes, the Greek mathematician who developed this method of finding prime numbers, was born in 276 B.C. in Cyrene, which is now in Libya in North Africa. He is the first person known to have calculated Earth's circumference.

Getting Ready

This activity is easiest to lead if everyone has pens or markers in the same four colors. The instructions assume that everyone has yellow, red, green, and blue, but you can adjust if you need to.

Before your group starts, make sure the members know the difference between odd numbers and even numbers. You should also discuss multiples. A multiple is what results when you multiply a number by other numbers. Some multiples of the number 2, for instance, are 4 (which is 2×2), 6 (which is 2×3), and 14 (which is 2×7).

The best way to present this activity is to demonstrate each step and have people follow along, step by step. We suggest that you provide a copy of *Hundreds Chart* (page 52) to each member of your group and lead the group through the activity using the *Eratosthenes' Sieve* instructions (page 47) as your guide. If you want people to work individually, you can give each a copy of the *Eratosthenes' Sieve* instructions as well.

Finding Patterns

Here are some of the patterns that group members may notice as they work through the activity:

- All multiples of 2 are even numbers.
- All multiples of 2 are lined up in vertical columns on the chart.
- Multiples of 3 form diagonals in the chart.
- Multiples of 5 are lined up in vertical columns under the numbers 5 and 10.
- Multiples of 7 form a pattern that resembles the way the knight moves in chess. Start with a multiple of 7, and move two rows down and one column to the right. There you'll find another multiple of 7.

Tricks to Remember

While marking their charts, people can learn a few tricks that will help them with multiplication:

- The trick to finding multiples of 2 is to look for even numbers.
- The trick to finding multiples of 3 is to add up the digits. If the sum is a multiple

of 3, then the original number is a multiple of 3, too.

- The trick to finding multiples of 5 is to look for numbers ending in 5 or zero.

Marking the Key

Make sure everyone updates the key at the bottom of the chart each time a new set of multiples is marked.

The key gets more interesting when people start marking multiples of composite numbers. (Remember, a *composite number* is a number found by multiplying two or more numbers together.) Once a chart is marked with all the multiples of 2 and 3, it's easy to find all the multiples of 6. Because 6 is equal to 2×3, any box that's marked as a multiple of 2 *and* a multiple of 3 is a multiple of 6.

If a number is a multiple of two prime numbers, it's also a multiple of those two numbers multiplied together!

Counting the Prime Numbers

After people have filled out the top row of the chart, all the remaining unmarked numbers are prime. Tell everyone to circle all the remaining numbers in pencil and count them.

People may get different answers. Sometimes they miss a multiple. For example, it's easy to overlook 91 as a multiple of 7. See if everyone can agree on the number of primes between 1 and 100. The correct number is 25.

Where's the Math?

Understanding prime and composite numbers helps people when they are working with fractions.

When students are adding or subtracting fractions, they often have to find a *common denominator*—a multiple that two numbers have in common. Suppose your group had to add $\frac{2}{5}$ and $\frac{1}{6}$. One look at the completed charts reveals that 30 is the smallest number that is a multiple of 5 and of 2 and 3 (or 6). So, they would know that 30 is a common denominator of 5 and 6.

Another common fraction operation is writing a fraction in its lowest terms. That means finding the greatest number that will divide evenly into both the *numerator* (the top part of the fraction) and the *denominator* (the bottom part of the fraction). For example, if you asked group members to write $\frac{12}{30}$ in lowest terms, they would need to find the greatest number that divides evenly into both 12 and 30. The completed chart shows that 12 and 30 are both multiples of 6. Dividing 6 into both the numerator and the denominator turns $\frac{12}{30}$ into $\frac{2}{5}$. By knowing which numbers are prime, they would know that $\frac{2}{7}$ and $\frac{12}{23}$ cannot be reduced.