# Mastering Matr Facts 

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For some students, mastering the basic math facts is a task that may need a little extra "something" to help them attach meaning to an abstract process. Some of the facts are learned quickly, while others seem to take forever to Gecome part of the student's knowledge base. On these pages, you will find strategies that can be usefulfor some of the students who are struggling to learn their math facts. Please keep in mind, however, that students do not all learn in the same way, and these strategies may not work equally well for all students.

## "Pair" the Task Down

Ulsing the student's knowledge of the Commutative Property, pair up the facts on the opposing sides of the "doubles diagonal." This will bring the total number of facts to be learned down to 55 instead of 100, a much less formidable task.

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All of the facts on one side of the "doubles line" have a
partner on the other side. Just turn the fact around, and you get the same answer.
Examples:
$5+3=8$
$3+5=8$
$3 \mathrm{X} 5=15$
$5 \times 3=15$

## Working Witf Zero

Adding, subtracting, multiplying, and dividing with a zero all have special properties that are easily learned.

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Adding zero to a number does not change the number. Subtracting a zero from a number does not change the number. Multiplying and dividing by a zero produces an answer of zero.
Examples:
$0+7=7$
$8-0=8$
$7 \mathrm{X} 0=0$
$5 \div 0=0$

## Ones-More, Less, the Same

Adding one to a number is simply counting on to the next number, subtracting one means going back. Multiplying or dividing by one leaves the answer the same.

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Adding one to a number is like counting:
$7+1=8$
Subtracting one from a number is
like counting backwards:
$9-1=8$
Multiplying or dividing by one has the other number as the answer:
$9 \div 1=9$
$7 \mathrm{X} 1=7$

## Doubles

When students begin working with doubles, they often need the experience of using manipulatives to see the results in concrete form. Later, as they can visualize the process and move to the abstract, doubles become easier and some of the first facts le arned.

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When adding doubles, try using manipulatives:
$4+4=8$
Subtracting doubles, the answer is always a zero:
$7-7=0$
Multiplying doubles is usually easier for students than other facts: $6 \mathrm{X} 6=36$
Dividing by doubles is similar to subtraction, the answer is one:
$9 \div 9=1$

## Doubles Plus One

This strategy can be used when one of the numbers is one more than the other. It works for addition and subtraction. In multiplication and division, the answer is also predictable, but not quite as simple as for addition and subtraction.

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Addition: Like doubles and
counting one more:
$4+5=(4+4)+1=9$
Subtraction, the answer is one:
$8-7=(1+7)-7=1+(7-7)$
Multiplication:
$6 \times 7=(6 \times 6)+6=36+6=42$
In division:
$56 \div 7=(49 \div 7)+(7 \div 7)$

$$
=7+1=8
$$

## Making Fives

Working with fives seems a natural with most students. Adding and subtracting fives in early stages needs the concrete examples manipulatives provide. Connect the experience to the fact that five is half of a group of 10. When multiplying and dividing fives, it is simply a form of counting forwards or backwards by fives.

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Addition: $3+5$ is five and three more, $7+5$ is $5+5+2$ more
Subtraction: regroup the 8 to make a five plus a three. $8-5$ is
$(5+3)-5=(5-5)+3=3$
Multiplication: all answers end in 5 or 0 as $7 \times 5=35$
In division: if a quotient ends in a five or a zero, five can be the divisor or the dividend as in $40 \div 8=5$ or $45 \div 5=9$

## Working With $\mathcal{N}$ (ines

Working with nines in addition and subtraction is like adding or subtracting by ten, and counting back one. Multiplication with nines is similar, but with a special twist - the sum of the digits in the product adds up to nine (numbers less than ten, and 18 in numbers over 10 (to 20). Dividing by nine, the answer is one more than the tens digit of the dividend.

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Addition: add ten and count back
one $5+9=5+(10-1)=$

$$
15-1=14
$$

Subtraction: take ten away and add one back $17-9=(17-10)+1=$

$$
7+1=8
$$

Multiplication: 4 X $9+36$,

$$
(3+6=9)
$$

Division: $54 \div 9=6$ (the tens digit
of the dividend is 5 , so $5+1=6$ )

## Making $\mathcal{T e n s , ~ a ~ S t r a t e g y ~ f o r ~} \mathcal{A d d i t i o n}$ and $S$ ubtraction

Teaching students to regroup numbers and parts of numbers to tens, makes mental math much easier for them. Work with students to learn which pairs add up to ten and how to recognize ones that are one or two more or one or two less, will help.

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Addition: combinations which add up to ten are easier $2+8=10$, and one more or less use counting on or back $3+8=11$, $(+1) ; 2+7=9$, (-1)
Subtraction: similar to addition, but using the inverse process
$11-8=3=$
$(11-8)=(11-1)-(8-1)=3$

Multiplication and Division by Ten, Ele ven, and Twe fve
For the intermediate students that need to learn the multiplication and division facts through twelve, there are some strategies that will assist them.

## Tens

Multiplying and dividing by ten is like counting by tens, or adding a zero to a factor $(11 X 10=110,7 \times 10=70)$. Dividing by ten uses the tens and fundred digits for a quotient $120 \div 10=12,80 \div 10=8$ ).
Ele vens
Multiplying and dividing by eleven is similar to ones and tens. When multiplying by eleven and the other factor is less than ten, double the smaller factor and you have the answer (5 $\times 11=55,9 \times 11=99)$. For eleven times eleven and eleven times twelve, think ten times the number and one times the number $\left(\begin{array}{ll}11 & X \\ 12\end{array}=\left(\begin{array}{lll}10 & X & 12\end{array}\right)+\left(\begin{array}{lll}1 & X & 12\end{array}\right)=132\right.$
Division is the opposite of this process. $121=120+11$ and $(120 \div 11)+$ $(11 \div 11)=10+1=11$
I we tves
For twelve, thinking in terms of a dozen may help, or simply the challenge of learning the hardest (largest) facts.

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## Iust LearnThem

Sometimes there are a few facts that just need to be memorized. Help students learn them by having a "Fact for the Day." Each time the child is spoken to, they must tell you the fact. They have to repeat it in order to leave the room for recess and to enter the room. By the end of the day, it should have been added to the list of the le arned facts.

## Make It Manageable

The most important thing teachers can do for their students is to make the mastery of basic facts a manageable task. Helping them make a grid of the facts that they know will help. Also, deciding which facts need to be learned next and helping them learn a feweach day. Begin working with the facts required for the grade level at the very beginning of the year. Work on learning and practicing facts long before taking the first test. In the intermediate grades, taking a practice test near the beginning of the year can felp pinpoint problems that may exist from previous years.

