

Mathematics Navigator

# Misconceptions and Errors



AMERICA'S CHOICE  
MATHEMATICS NAVIGATOR™



## In this Guide

Misconceptions and errors are addressed as follows:

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## Place Value

1. When counting tens and ones (or hundreds, tens, and ones), the student misapplies the procedure for counting on and treats tens and ones (or hundreds, tens, and ones) as separate numbers.

### Example

When asked to count collections of bundled tens and ones, such as  $||| \bullet\bullet$ , student counts 10, 20, 30, 1, 2, instead of 10, 20, 30, 31, 32.

2. The student has alternative conception of multidigit numbers and sees them as numbers independent of place value.

### Example

Student reads the number 32 as “thirty-two” and can count out 32 objects to demonstrate the value of the number, but when asked to write the number in expanded form, she writes “ $3 + 2$ .”

Student reads the number 32 as “thirty-two” and can count out 32 objects to demonstrate the value of the number, but when asked the value of the digits in the number, she responds that the values are “3” and “2.”

3. The student recognizes simple multidigit numbers, such as thirty (30) or 400 (four hundred), but she does not understand that the position of a digit determines its value.

### Example

Student mistakes the numeral 306 for thirty-six.

Student writes 4008 when asked to record four hundred eight.

4. The student misapplies the rule for reading numbers from left to right.

### Example

Student reads 81 as eighteen. The teen numbers often cause this difficulty.

5. The student orders numbers based on the value of the digits, instead of place value.

Example

$69 > 102$ , because 6 and 9 are bigger than 1 and 2.

6. The student undergeneralizes results of multiplication by powers of 10 and does not understand that shifting digits to higher place values is like multiplying by powers of 10.

Example

When asked to solve a problem like  $\square \times 36 = 3600$ , the student either divides or cannot respond.

7. Student has limited his understanding of numbers to one or two representations.

Example

Student may be able to read and write the number 4,302,870 in standard form but he does not link this number to a representation using tally marks in a place value chart or to expanded form.

8. Student applies the alternate conception “Write the numbers you hear” when writing numbers in standard form given the number in words.

Example

When asked to write the number five hundred eleven thousand in standard form, the student writes 500,11,000 with or without commas.

Example

When asked to write the number sixty-two hundredths, student writes 62.00 or 6200.

9. Student misapplies the rule for “rounding down” and actually lowers the value of the digit in the designated place.

Example

When asked to round to the nearest ten thousand, student rounds the number 762,398 to 750,000 or 752,398.

Example

When asked to round to the nearest tenth, student rounds the number 62.31 to 62.2 or 62.21.

- 10.** Student misapplies the rule for “rounding up” and changes the digit in the designated place while leaving digits in smaller places as they are.

Example

Student rounds 127,884 to 128,884 (nearest thousand).

Example

Student rounds 62.38 to 62.48 (nearest tenth).

- 11.** Student overgeneralizes that the comma in a number means “say thousands” or “new number.”

Example

Student reads the number 3,450,207 as “three thousand four hundred fifty thousand two hundred seven.”

Example

Student reads the number 3,450,207 as “three, four hundred fifty, two hundred seven.”

- 12.** Student lacks the concept that 10 in any position (place) makes one (group) in the next position and vice versa.

Example

If shown a collection of 12 hundreds, 2 tens, and 13 ones, the student writes 12213, possibly squeezing the 2 and the 13 together or separating the three numbers with some space.

Example

$0.72 + 0.72 = 0.144$  or  $\begin{array}{r} 32,871 \\ + 9,324 \\ \hline 311,195 \end{array}$  or 3,111,195

- 13.** Student lacks the concept that the value of any digit in a number is a combination of the face value of the digit and the place.

Example

When asked the value of the digit 8 in the number 18,342,092 the student responds with “8” or “one million” instead of “eight million.”

# Addition and Subtraction

1. The student has overspecialized his knowledge of addition or subtraction facts and restricted it to “fact tests” or one particular problem format.

## Example

Student completes addition or subtraction facts assessments satisfactorily but does not apply the knowledge to other arithmetic and problem-solving situations.

2. The student may know the commutative property of addition but fails to apply it to simplify the “work” of addition or misapplies it in subtraction situations.

## Example

Student states that  $9 + 4 = 13$  with relative ease, but struggles to find the sum of  $4 + 9$ .

Student writes (or says) “ $12 - 50$ ” when he means  $50 - 12$ .

3. Thinking that subtraction is commutative, for example  $5 - 3 = 3 - 5$

## Example

$$5 - 3 = 3 - 5$$

4. The student may know the associative property of addition but fails to apply it to simplify the “work” of addition.

## Example

Student labors to find the sum of three or more numbers, such as  $4 + 7 + 6$ , using a rote procedure, because she fails to recognize that it is much easier to add the numbers in a different order.



5. The student tries to overgeneralize immature addition or subtraction methods, instead of developing more effective methods.

**Example**

Student may have learned the early childhood method of “recount all” and stopped there. When the numbers get too big to recount, she has nothing else to draw on.

6. The student may be unable to generalize methods that he already knows for adding and subtracting to a new situation.

**Example**

Student may be perfectly comfortable with addition facts, such as  $6 + 7$ , but he is does not know how to extend this fact knowledge to a problem, such as  $16 + 7$ .

7. The student has overspecialized during the learning process so that she recognizes some addition and/or subtraction situations as addition or subtraction but fails to classify other situations appropriately.

**Example**

Student recognizes that if there are 7 birds in a bush and 3 fly away, you can subtract to find out how many are left.

However, she may be unable to solve a problem that involves the comparison of two amounts or the missing part of a whole.

8. The student knows *how* to add but does not know *when* to add (other than because he was told to do so, or because the computation was written as an addition problem).

**Example**

Student cannot explain why he should add or connect addition to actions with materials.

9. The student knows *how* to subtract but does not know *when* to subtract (other than because she was told to do so, or because the computation was written as a subtraction problem).

Example

Student cannot explain why she should subtract or connect subtraction to actions with materials.

10. The student has overspecialized during the learning process so that he recognizes some addition situations as addition but fails to classify other addition situations appropriately.

Example

Student recognizes that if it is  $47^{\circ}$  at 8 AM, and the temperature rises by  $12^{\circ}$  between 8 AM and noon, you add to find the temperature at noon.

However, he then states that the situation in which you know that the temperature at 8 AM was  $47^{\circ}$  and that it was  $12^{\circ}$  cooler than it is now is not addition.

11. The student has overspecialized during the learning process so that she recognizes some subtraction situations as subtraction but fails to classify other subtraction situations appropriately.

Example

Student recognizes that if there are 7 birds in a bush and 3 fly away, you can subtract to find out how many are left.

However, she may be unable to solve a problem that involves the comparison of two amounts or the missing part of a whole.

12. The student can solve problems as long as they fit one of the following “formulas.”

$$a + b = ?$$

$$\begin{array}{r} a \\ b \\ + c \end{array}$$

$$b - a = ?$$

$$\begin{array}{r} b \\ - a \end{array}$$

He has over-restricted the definition of addition and/or subtraction.

**Example**

Given any other situation, the student responds, “You can’t do it,” or resorts to “guess and check.”

13. The student sees addition and subtraction as discrete and separate operations. Her conception of the operations does not include the fact that they are linked as inverse operations.

**Example**

Student has difficulty mastering subtraction facts because she does not link them to addition facts. She may know that  $6 + 7 = 13$  but fails to realize that this fact also tells her that  $13 - 7 = 6$ .

Student can add  $36 + 16 = 52$  but cannot use addition to help estimate a difference, such as  $52 - 36$ , or check the difference once it has been computed.

14. When adding or subtracting, the student misapplies the procedure for regrouping.

**Example**

$$\begin{array}{r} \phantom{0}^1 \phantom{0}^1 \phantom{0}^1 \phantom{0}^1 \\ 63,842 \\ + 24,036 \\ \hline 98,888 \end{array}$$

15. When subtracting, the student overgeneralizes from previous learning and “subtracts the smaller number from the larger one” digit by digit.

**Example**

$$\begin{array}{r} 62,483 \\ - 58,575 \\ \hline 16,112 \end{array}$$

# Multiplication and Division

1. The student has overspecialized his knowledge of multiplication or division facts and restricted it to “fact tests” or one particular problem format.

## Example

Student completes multiplication or division facts assessments satisfactorily but does not apply the knowledge to other arithmetic and problem-solving situations.

2. The student may know the commutative property of multiplication but fails to apply it to simplify the “work” of multiplication.

## Example

Student states that  $9 \times 4 = 36$  with relative ease, but struggles to find the product of  $4 \times 9$ .

3. The student may know the associative property of multiplication but fails to apply it to simplify the “work” of multiplication.

## Example

Student labors to find the product of three or more numbers, such as  $8 \times 13 \times 5$ , because he fails to recognize that it is much easier to multiply the numbers in a different order.

4. The student sees multiplication and division as discrete and separate operations. His conception of the operations does not include the fact that they are linked as inverse operations.

## Example

Student has reasonable facility with multiplication facts but cannot master division facts. He may know that  $6 \times 7 = 42$  but fails to realize that this fact also tells him that  $42 \div 7 = 6$ .

Student knows procedures for dividing but has no idea how to check the reasonableness of his answers.

5. The student has overspecialized during the learning process so that she recognizes some multiplication and/or division situations as multiplication or division and fails to classify others appropriately.

Example

Student recognizes that a problem in which 4 children share 24 grapes is a division situation but states that a problem in which 24 cherries are distributed to children by giving 3 cherries to each child is not.

Example

Student recognizes “groups of” problems as multiplication but does not know how to solve scale, rate, or combination problems.

6. The student knows *how* to multiply but does not know *when* to multiply (other than because he was told to do so, or because the computation was written as a multiplication problem).

Example

Student cannot explain why he should multiply or connect multiplication to actions with materials.

7. The student knows *how* to divide but does not know *when* to divide (other than because she was told to do so, or because the computation was written as a division problem).

Example

Student cannot explain why she should divide or connect division to actions with materials.

8. The student does not understand the distributive property and does not know how to apply it to simplify the “work” of multiplication.

Example

Student has reasonable facility with multiplication facts but cannot multiply  $12 \times 8$  or  $23 \times 6$ .

9. The student applies a procedure that results in remainders that are expressed as “R#” or “remainder #” for all situations, even those for which such a result does not make sense.

Example

When asked to solve the following problem, student responds with an answer of “10 R2 canoes,” even though this makes no sense:

There are 32 students attending the class canoe trip. They plan to have 3 students in each canoe.

How many canoes will they need so that everyone can participate?

10. The student sees multiplication and division as discrete and separate operations. His conception of the operations does not include the fact that they are linked as inverse operations.

Example

Student has reasonable facility with multiplication facts but cannot master division facts. He may know that  $6 \times 7 = 42$  but fails to realize that this fact also tells him that  $42 \div 7 = 6$ .

Student knows procedures for dividing but has no idea how to check the reasonableness of his answers.

11. The student undergeneralizes the results of multiplication by powers of 10. To find products like  $3 \times 50 = 150$  or  $30 \times 50 = 1,500$ , she must “work the product out” using a long method of computation.

Example

$$\begin{array}{r}
 300 \\
 \times 500 \\
 \hline
 000 \\
 0000 \\
 + 150000 \\
 \hline
 150000
 \end{array}$$

- 12.** The student can state and give examples of properties of multiplication but does not apply them to simplify computations.

**Example**

The student multiplies  $6 \times 12$  with relative ease but struggles to find the product  $12 \times 6$ .

or

The student labors to find the product  $12 \times 15$  because he does not realize that he could instead perform the equivalent but much easier computation,  $6 \times 30$ .

or

The student has reasonable facility with multiplication facts but cannot multiply  $6 \times 23$ .

- 13.** The student misapplies the procedure for multiplying multidigit numbers by ignoring place value.

**Example**

- a. Multiplies correctly by ones digit but ignores the fact that the 3 in the tens place means 30. Thus,  $30 \times 60 = 1,800$ .

$$\begin{array}{r} 60 \\ \times 38 \\ \hline 480 \\ + 180 \\ \hline 660 \end{array}$$

- b. Multiplies each digit as if it represented a number of "ones." Ignores place value completely.

$$\begin{array}{r} 47 \\ \times 52 \\ \hline 14 \\ 8 \\ 35 \\ + 20 \\ \hline 77 \end{array}$$

14. The student misapplies the procedure for regrouping as follows:

The first step (multiplying by ones) is done correctly, but the same numbers are used for regrouping again when multiplying by 10s whether it is appropriate or not.

Example

$$\begin{array}{r} \overset{3}{3}7 \\ \times 65 \\ \hline 185 \\ + 2120 \\ \hline 2305 \end{array}$$

$$\begin{array}{r} \overset{1}{1}28 \\ \times 75 \\ \hline 640 \\ + 8860 \\ \hline 9500 \end{array}$$

15. The student overgeneralizes the procedure learned for addition and applies it to multidigit multiplication inappropriately.

Original process for addition: When performing addition with regrouping, the student first adds the amount that is regrouped to the appropriate amount in the topmost addend and then continues by adding the remaining amounts in that place value column.

Inappropriate generalization: When performing multiplication, the student first adds the amount that is regrouped to the amount in the multiplicand and then multiplies (instead of multiplying first and then adding the amount that was regrouped).

Example

Students regroup the 4 tens. They then add 4 to 0 to get 4 and multiply that by 8 ( $4 \cdot 8$ ) to get 32, instead of multiplying  $8 \cdot 0$  and then adding the 4, to get 4.

$$\begin{array}{r} \overset{3}{3}\overset{4}{0}6 \\ \times 18 \\ \hline 4028 \\ + 2060 \\ \hline 6088 \end{array}$$

16. The student generalizes what she learned about single-digit multiplication and applies it to multidigit multiplication by multiplying each column as a separate single-digit multiplication. This can also be looked at as an example of Misconception 5.

Example

$$\begin{array}{r} 34 \\ \times 62 \\ \hline 188 \end{array}$$

$$\begin{array}{r} \overset{1}{1}28 \\ \times 71 \\ \hline 848 \end{array}$$



17. Thinking that division is commutative, for example  $5 \div 3 = 3 \div 5$

Example

$$5 \div 3 = 3 \div 5$$

18. Thinking that dividing always gives a smaller number
19. Thinking that multiplying always gives a larger number
20. Always dividing the larger number into the smaller

Example

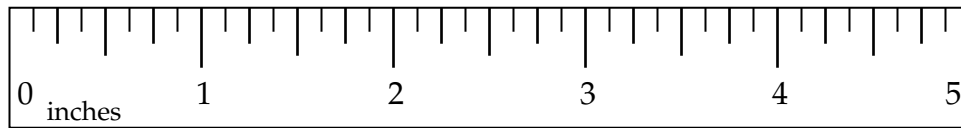
$$4 \div 8 = 2$$

21. Thinking that the operation that needs to be performed (+, -, ×, ÷) is defined by the numbers in the problem

# Fractions

1. Student has restricted his definition of fractional parts on the ruler so that he thinks that an inch is the specific distance from 0 to 1 and does not understand that an inch unit of length is an inch, anywhere on the ruler.

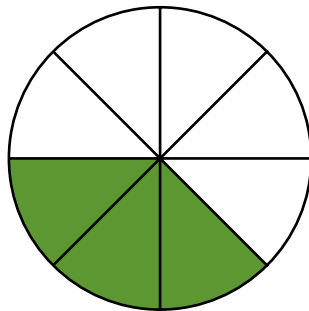
Example



Student says that the line segment is  $3\frac{1}{2}$ " or that you cannot tell how long the green bar is.

2. Student writes fraction as part/part instead of part/whole.

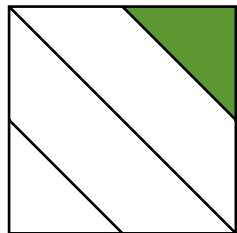
Example



Student says that  $\frac{3}{5}$  are shaded.

3. Student does not understand that when finding fractions of amounts, lengths, or areas, the parts need to be equal in size.

Example

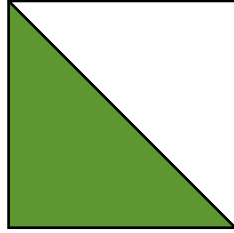


Student says that  $\frac{1}{4}$  of the square is shaded.

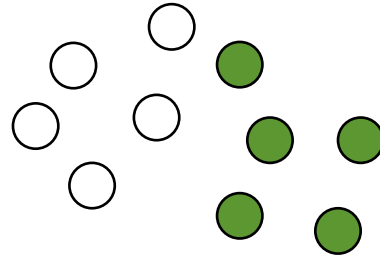
4. Student does not understand that fractions are numbers as well as portions of a whole.

Example

Student recognizes  $\frac{1}{2}$  in situations like these:



one-half of the area is shaded



one-half of the circles are shaded

but cannot locate the number  $\frac{1}{2}$  on a number line, or says that “one-half is not a number, it is a part.”

5. Student thinks that mixed numbers are larger than improper fractions because mixed numbers contain a whole number part and whole numbers are larger than fractions.

Example

Student says that  $1\frac{4}{5} > \frac{9}{5}$  because whole numbers are larger than fractions.

6. Student is confused about the whole in complex situations.

Example

Anna spent  $\frac{3}{4}$  of her homework time doing math. She still has  $\frac{1}{2}$  hour of homework left to do. What is the total time Anna planned for homework?

When looking at this problem, the student can easily become confused about the whole. Is the whole the total time Anna planned for homework, or is the whole one hour?

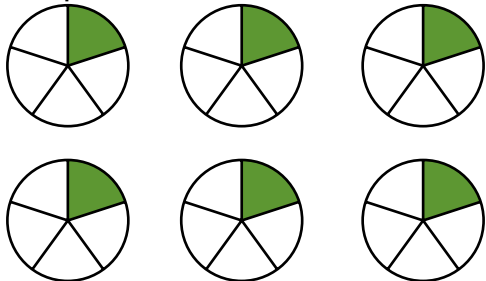
7. Student has restricted her definition and thinks fractions have to be less than 1.

Example

When confronted with an improper fraction, the student says it is not a fraction because in a fraction the numerator is always less than the denominator.

8. Student counts pieces without concern for whole.

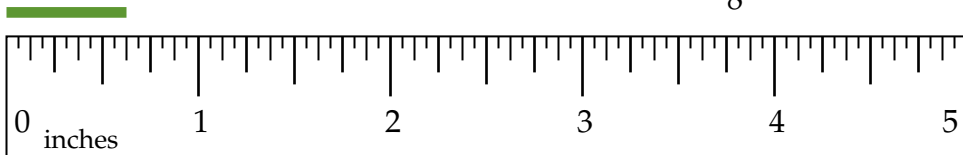
Example



Student says that  $\frac{6}{30}$  of a circle is shaded.

Example


When asked to measure the line segment to the nearest  $\frac{1}{8}$  inch,



student says that the line segment is  $\frac{10}{8}$  inches in length.

9. Student thinks that when finding fractions using area models, the equal-sized pieces must look the same.

Example



Student says this diagram does not show fourths of the area of the square because the pieces are “not the same (shape).”

10. Student overgeneralizes and thinks that “all  $\frac{1}{4}$ s (for example) are equal”; she does not understand that the size of the whole determines the size of the fractional part.

Example

Amir and Tamika both went for hikes. Amir hiked 2 miles and Tamika hiked 8 miles.

Student thinks that when both students had completed  $\frac{1}{4}$  of their hikes, they have each walked the same distance because  $\frac{1}{4} = \frac{1}{4}$ .

- 11.** Student has restricted his definition of fractions to one type of situation or model, such as part/whole with pieces.

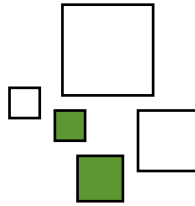
**Example**

Student does not recognize fractions as points on a number line or as division calculations.

- 12.** Student overgeneralizes from experiences with fractions of amounts, lengths, or areas and thinks that when dealing with a fraction of a set, parts always have to be equal in size.

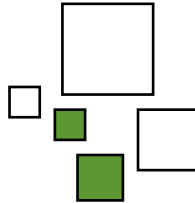
**Example**

What fraction of the squares is shaded?



Student says, "This is not a fraction because the parts are not equal."

**Example**



When given this diagram as one of several possible choices for  $\frac{2}{5}$ , student fails to identify the example as  $\frac{2}{5}$ .

- 13.** Overgeneralizes fraction notation or decimal notation and confuses the two

**Example**

$$\frac{1}{4} = 1.4 \text{ or } \frac{1}{4} = 0.4$$

- 14.** Misapplies rules for comparing whole numbers in fraction situations

**Example**

$\frac{1}{8}$  is bigger than  $\frac{1}{6}$  because 8 is bigger than 6

15. Overgeneralizes the idea that “the bigger the denominator, the smaller the part” by ignoring numerators when comparing fractions

Example

$$\frac{1}{4} > \frac{3}{5} \text{ because fourths are greater than fifths}$$

18. Restricts interpretation of fractions inappropriately and does not understand that different fractions that name the same amount are equivalent

Example

$$\frac{2}{3} \text{ and } \frac{4}{6} \text{ cannot name the same amount because they are different fractions}$$

19. Misapplies additive ideas when finding equivalent fractions

Example

$$\frac{3}{8} = \frac{4}{9} \text{ because } 3 + 1 = 4 \text{ and } 8 + 1 = 9$$

20. Overgeneralizes results of previous experiences with fractions and associates a specific number with each numerator or denominator when simplifying fractions

Example

Prime numbers like 2, 3, or 5 always become 1 when you simplify and even numbers are always changed to one-half of their value. Using “rules” like this the student gets correct answers some of the time, like

$$\frac{2}{8} = \frac{1}{4} \text{ and } \frac{4}{6} = \frac{2}{3},$$

but not all the time.

The student ignores the fact that some of the fractions are already in simplest form.

23. Student knows only a limited number of models for interpreting fractions

Example

Student does not recognize fractions as points on a number line or as division calculations.

- 21.** When adding fractions, generalizes the procedure for multiplication of fractions by adding the numerators and adding the denominators

Example

$$\frac{1}{4} + \frac{1}{4} = \frac{2}{8}$$

Note that this error can also be caused by the alternative “conception” that fractions are just two whole numbers that can be treated separately.

- 22.** Students do not use benchmark numbers like 0,  $\frac{1}{2}$ , and 1 to compare fractions because they have restricted their understanding of fractions to part-whole situations and do not think of the fractions as numbers.

Example

When asked to compare two fractions like  $\frac{7}{12}$  and  $\frac{5}{13}$  students cannot do so, start cutting fraction pieces, resort to guessing, or perform difficult computations (to find the decimal equivalents or common denominators) instead of comparing both numbers to one-half.

- 25.** Thinking that dividing by one-half is the same as dividing in half

Example

$$4 \div \frac{1}{2} = 2$$

- 26.** When dividing a fraction by a whole number, divide the denominator by the whole number

Example

$$\frac{1}{26} \div 2 = \frac{1}{13}$$

- 27.** Confusing which number is divided into or multiplied by which; dividing the second number by the first

Example

$$\frac{1}{4} \div \frac{1}{8} = \frac{1}{2}$$

- 28.** When multiplying a fraction by a fraction, dividing both the numerator and denominator of one fraction by the denominator of the other

Example

$$\frac{1}{2} \times \frac{4}{6} = \frac{2}{3}$$

- 29.** Using the numerator and ignoring the denominator

Example

When asked to find  $\frac{2}{3}$  of 9 objects, finding 2 objects out of 9

- 30.** Thinking that the denominator is always the number of objects, even if the fraction has been reduced

Example

Reading  $\frac{3}{4}$  of 8 objects as  $\frac{3}{8}$

- 31.** When writing a fraction, comparing two parts to each other rather than comparing one part to the whole
- 32.** Thinking that decimals are bigger than fractions because fractions are really small things
- 33.** Thinking that you cannot convert a fraction to a decimal—that they can not be compared
- 34.** Thinking that doubling the size of the denominator doubles the size of the fraction
- 35.** Thinking that multiplying the numerator and the denominator by the same number increases the value of the fraction
- 36.** Thinking that dividing the numerator and the denominator by the same number reduces the value of the fraction
- 37.** When adding two fractions, adding the numerators and multiplying the denominators

Example

$$\frac{3}{5} + \frac{1}{2} = \frac{4}{10}$$



- 38.** When adding two fractions, adding the numerators and multiplying the denominators
- 39.** When subtracting mixed numbers, always subtracting the smaller whole number from the larger whole number and subtracting the smaller fraction from the larger fraction
- 40.** When multiplying fractions, multiplying the numerator of the first fraction by the denominator of the second, and adding the product of the denominator of the first and the numerator of the second

Example

$$\frac{3}{4} \times \frac{5}{7} = (3 \times 7) + (4 \times 5) = 41$$

- 41.** When multiplying fractions, using the “invert and multiply” procedure by inverting the second fraction and multiplying
- 42.** When dividing fractions, dividing the numerators and dividing the denominators;

Example

$$\frac{6}{7} \div \frac{2}{7} = \frac{3}{1}$$

# Decimals

1. Student misapplies knowledge of whole numbers when reading decimals and ignores the decimal point.

**Example**

Student reads the number 45.7 as, “four fifty-seven” or “four hundred fifty-seven.”

2. Student misapplies procedure for rounding whole numbers when rounding decimals and rounds to the nearest ten instead of the nearest tenth, etc.

**Example**

Round 3045.26 to the nearest tenth. Student responds, “3050” or “3050.26”

3. Student misapplies rules for comparing whole numbers in decimal situations.

**Example**

$0.058 > 0.21$  because  $58 > 21$

**Example**

$2.04 > 2.5$  because it has more digits

3. Reading the marks on the ruler as whole numbers
4. Mixing decimals and fractions when reading decimal numbers
6. Thinking that a decimal is just two ordinary numbers separated by a dot

- The decimal point in money separates the dollars from the cents

**Example**

100 cents is \$0.100

- The decimal point is used to separate units of measure

**Example**

.6 meters is 1 meter 6 centimeters

7. When adding a sequence, adding the decimal part separately from the whole number part

**Example**

0.50, 0.75, 0.100 rather than 0.50, 0.75, 1.00

8. Verbalizing decimals as whole numbers without the place value designated;

Example

“point ten” instead of “point one zero,”

“point twenty-five” instead of “twenty-five hundredths” or “point two five”

- 9.. Adding or subtracting without considering place value, or starting at the right as with whole numbers

Example

$$4.15 + 0.1 = 34.16 \text{ or } 12 - 0.1 = 11$$

10. Believing that two decimals can always be compared by looking at their “lengths”

Example

“longer numbers are always bigger,” or “shorter numbers are always bigger”

11. Misunderstanding the use of zero as a placeholder

Example

1.5 is the same as 1.05

12. Thinking that decimals with more digits are smaller because tenths are bigger than hundredths and thousandths

Example

.845 is smaller than .5

13. Thinking that decimals with more digits are larger because they have more numbers

Example

1,234 is larger than 34 so 0.1234 is larger than 0.34

14. Mistakenly applying what they know about fractions;

Example

$$\frac{1}{204} > \frac{1}{240}, \text{ so } 0.204 > 0.240$$

15. Mistakenly applying what they know about whole numbers

Example

$600 > 6$ , so  $0.600 > 0.6$

16. Believing that zeros placed to the right of the decimal number changes the value of the number

Example

0.4 is smaller than 0.400 because 4 is smaller than 400, or 0.81 is closer to 0.85 than 0.81 is to 0.8

17. Believing that a number that has only tenths is larger than a number that has thousandths

Example

$0.5 > 0.936$  because 0.936 has thousandths and 0.5 has only tenths

18. When multiplying by a power of ten, multiplying both sides of the decimal point by the power of ten  
When dividing by a power of ten, dividing both sides of the decimal point by the power of ten

Example

$$6.9 \times 10 = 60.90$$

$$70.5 \div 10 = 7\frac{1}{2}$$

19. Not using zero as a placeholder when ordering numbers or finding numbers between given decimals that have different numbers of significant digits

Example

There are no numbers between 2.2 and 2.18

20. Not recognizing the denseness of decimals; for example, there are no numbers between 3.41 and 3.42

Example

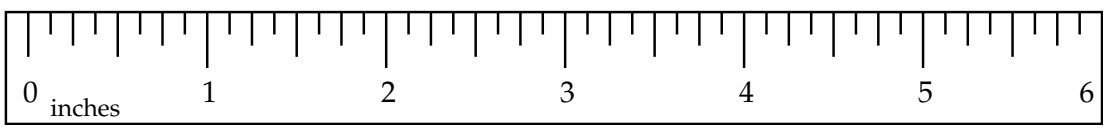
There are no numbers between 3.41 and 3.42

There is a finite number of expressions that will add or subtract to get a given decimal number

# Measurement

1. Student begins measuring at the end of the ruler instead of at zero.

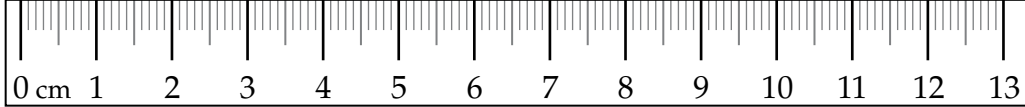
Example



Student response: line segment is  $2\frac{7}{8}$  inches in length

2. When measuring with a ruler, student counts the lines instead of the spaces.


Example



Student response: line segment is 6 centimeters in length

3. Student begins measuring at the number 1 instead of at zero and does not compensate.

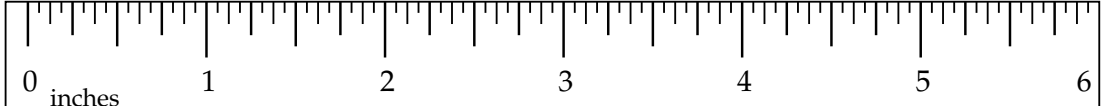
Example



Student response: line segment is 3 inches in length

4. Student counts intervals shown on the ruler as the desired interval regardless of their actual value.

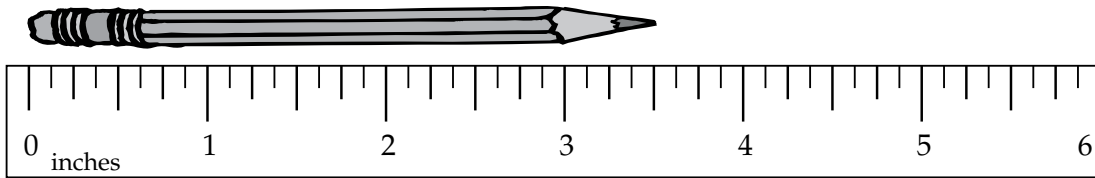
Example



Student response: line segment is  $\frac{10}{8}$  inches in length

5. Student fails to interpret interval marks appropriately.

Example



When asked to measure the pencil to the nearest  $\frac{1}{8}$  inch, the student responds with  $3\frac{3}{8}$  inches or  $3\frac{5}{8}$  inches because he fails to interpret the  $\frac{1}{2}$  inch mark as a  $\frac{1}{8}$  inch mark.

6. Student tries to use the formula for finding the perimeter of rectangular shapes on nonrectangular shapes.

Example



Student measures “length” (horizontal distance across) and “width” (vertical distance), then calculates perimeter as  $2 \times \text{length} + 2 \times \text{width}$ .

7. Student confuses area and perimeter.

Example

When asked to find the area of a rectangle with dimensions of  $12 \text{ cm} \times 4 \text{ cm}$ , the student adds  $12 + 4 + 12 + 4 = 32 \text{ cm}$ .

Example

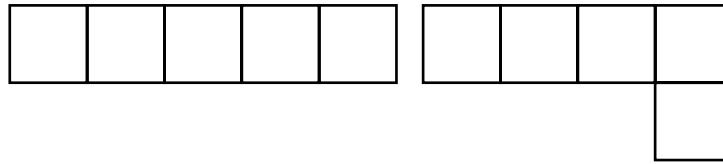
When asked to find the perimeter of a rectangle with dimensions of  $8 \text{ inches} \times 7 \text{ inches}$ , the student multiplies  $8 \times 7 = 56 \text{ inches}$  (or square inches).

Example

Student thinks that perimeter is the sum of the length and the width because area is length times width.

8. Student thinks that all shapes with a given perimeter have the same area or that all shapes with a given area have the same perimeter.

Example

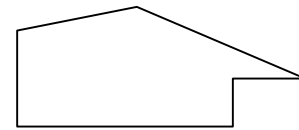


Since both of these shapes have an area of 5 square units and a perimeter of 12 units, the student concludes that all shapes with an area of 5 square units have a perimeter of 12 units or that all shapes with a perimeter of 12 units have an area of 5 square units.

9. Student tries to use the formula for finding the area of rectangular shapes on nonrectangular shapes.

Example

Student measures “length” (horizontal distance across) and “width” (vertical distance), then calculates area as length  $\times$  width.



10. Student may overgeneralize or undergeneralize the definition of area and/or perimeter situations.

Example

Student interprets all “wall painting” problems as area, even if the problem talks about the length of a striped border that is painted around the room.

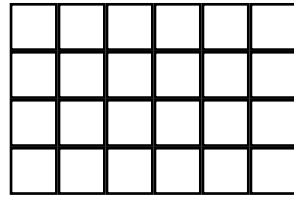
Example

Student interprets all “fence” problems as perimeter, even if the problem talks about the size of the garden that the fence encloses.

11. When counting perimeter of dimensions of shapes drawn on a grid, student counts the number of squares in the border instead of the edges of the squares.

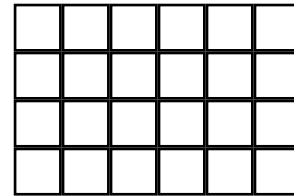
Example

When asked to find the perimeter of this rectangle, the student responds with 16 or 16 units or 16 squares.



Example

When asked to sketch a 4 cm × 5 cm rectangle, student sketches a 4 cm × 6 cm rectangle.



12. Student overgeneralizes base-10 and applies it to measurements inappropriately.

Example

When asked to change 1 hour 15 minutes to minutes, the student responds with 115 minutes or with 25 minutes.

Example

When asked to change 1 hour 15 minutes to hours, the student responds with 1.15 hours.

13. Believing that the size of a picture determines the size of the object in real life



- 14.** Student has a limited number of units of measure that he knows and understands and uses these units inappropriately.

Example

Student uses wrong notation or labels.

Example

Student chooses inappropriate unit of measure or inappropriate measuring tool for task.

Example

When faced with a unit he does not know, the student ignores the unit, guesses, or does nothing.

- 15.** Student does not understand elapsed time.

Example

Student can read a clock or a calendar but does not apply this knowledge to elapsed time problems.

Example

When faced with an elapsed time problem, student guesses or does nothing.

- 16.** Student lacks “benchmarks” that allow her to estimate measures.

Example

When faced with a problem that asks her to estimate a measurement, student guesses or does nothing.

# Percents

1. Not understanding that percents are a number out of one hundred; percents refer to hundredths
2. Confusing tenths with hundredths

Example

Writing 0.4 as 4% rather than 40%

3. Thinking percents cannot be greater than 100

Example

Writing 1.45 as .145%

4. Not realizing that one whole equals 100%
5. Not knowing which operation to use when working with percents
6. Having difficulty identifying the “whole” that the percent refers to
7. Thinking an increase of  $n\%$  followed by a decrease of  $n\%$  restores the amount to its original value.
8. Lack of understanding that percent increase has a multiplicative structure
9. Having difficulty using zero as a placeholder when writing a percent as a decimal

Example

Writing 6% as 0.6

10. Finding the increase or decrease instead of the final amount
11. Treating percents as though they are just quantities that may be added like ordinary discount amounts
12. Not recognizing the “whole” the percent refers to, and that a second percent change refers to a different “whole” than the first

# Functions and Graphs

1. Confusing the two axes of a graph
2. Not understanding the meaning of points in the same position relative to one of the axes
3. Thinking that the points on the graph stay in the same position even if the axes change
4. Thinking that graphs are “pictures” of situations, rather than abstract representations

## Example

Thinking that a speed graph of a bicycle coasting downhill and then uphill resembles the hill, first going down and then up;  
 a graph with negative slope means the object is falling;  
 if the graph is rising, the object is moving upward; or if the graph changes direction, the object changes direction. if two lines on a graph cross, the paths of the objects cross

5. Thinking that graphs always go through (or begin at) the origin
6. Thinking that graphs always cross both axes
7. Focusing on some attributes of a situation and ignoring others

## Example

Noting the existence of local minima but ignoring their relative positions or values

8. Reading the  $y$ -axis as speed even when it represents a different parameter
9. Thinking that the greatest numbers labeled on the axes represent the greatest values reached

## Example

If a graph of a race has the distance axis labeled up to 120 meters, the race is for 120 meters (even if it is a 100-meter race).

10. Thinking that all sequences are linear or increasing and linear
11. Not discriminating between linear and non-linear sequences
12. Not discriminating between increasing and decreasing sequences

13. Not discriminating between linear and proportional sequences
14. Thinking that linear problems are always proportional—for example, believing that if  $y = ax + b$  then doubling  $x$  will double  $y$ .

**Example**

Believing that a stack of ten nested cups will be twice the height of five nested cups.

15. Thinking that  $n \cdot n = 2n$
16. Confusing a table showing an actual situation (for example, 4 teams in a tournament) with a table that represents all of the games in the tournament
17. Being unable to generalize the  $n$ th case; only working with real numbers
18. Always trying to work a problem from the visual to the equation
19. Thinking that the results are random; there is no pattern
20. Trying to substitute numbers rather than writing a formula for each sequence

# Expression and Equations

1. Solving problems from left to right no matter what the operations are
2. Dividing the whole expression by the denominator rather than just the part that is the fraction
3. Disregarding exponents when calculating expressions

Example

$$2 \cdot 4^3 = 8$$

4. Multiplying by an exponent rather than multiplying the expression by itself

Example

$$6^2 = 6 \cdot 2, \text{ reading "6 squared" as "6 doubled"}$$

5. Not using standard algebraic conventions; writing expressions as in arithmetic

Example

$$x \cdot 5 = 1 \text{ rather than } 5x = 1$$

6. Not using parentheses when they are necessary to interpret the expression

Example

$$5 + x \cdot 5 \text{ rather than } 5(x + 5)$$

7. Not using standard algebraic conventions for exponents; writing expressions as in arithmetic

Example

$$x \cdot x \text{ rather than } x^2$$

- 8.** When simplifying expressions, writing like terms next to each other but not adding

Example

$$4 + 2x + 6 + x = 10 + 2x + x$$

- 9.** Adding a constant to a variable term

Example

$$10 + 2x = 12x$$

- 10.** Adding unlike terms

Example

$$10x^2 + 2x = 12x^2$$

- 11.** Not distributing multiplication to all terms in the parentheses (misusing the distributive property)

Example

$$2(x + 6) = 2x + 6$$

- 12.** Distributing multiplication by a negative term (or subtraction) to only the first term in an expression

Example

$$x - 2(x + 6) = x - 2x + 12 = -x + 12$$

- 13.** When simplifying expressions, writing like terms next to each other but not adding

Example

$$4 + 2x + 6 + x = 10 + 2x + x$$

14. Reading the equality sign as “makes” without considering what is on the other side of the equation

Example

$$9 + 10 = x + 9 \text{ as } 9 + 10 = 19$$

15. Confusing negative signs when adding and subtracting terms.

Example

$$2x + 12 = x \text{ as } x = 12 \text{ rather than } x = -12$$

16. Thinking that a variable can only stand for one particular number

17. Thinking that different variables must stand for different numbers

Example

$$x + 5 \neq y + 5 \text{ because } x \text{ and } y \text{ cannot be the same number}$$

18. Thinking that a variable represents an object rather than a number

Example

“If there are  $d$  days in  $w$  weeks, then  $w = 7d$  because a week equals seven days.” This interpretation is incorrect because  $w$  and  $d$  are identified as the “objects” week and day, rather than as the numbers of weeks and days. A correct equation would be  $d = 7w$ , because we would have to multiply the number of weeks by seven to get the number of days.

19. Thinking that a variable represents an object rather than a number

Example

“If there are  $d$  days in  $w$  weeks, then  $w = 7d$  because a week equals seven days.”

