Multiplication Algorithms

1. Partial Products Algorithm:

	67
	x 53
50 x 60	3000
50 x 7	350
3 x 60	180
3 x 7	+ 21
	3551

1. Partial Products Algorithm

The partial products algorithm for multiplication is based on the distributive, or grouping, property of multiplication. In this multiplication algorithm, each factor is thought as a sum of ones, tens, hundreds, and so on. For example, in 67 * 53, think 67 as 60 + 7, and 53 as 50 + 3. Then each part of one factor is multiplied by each part of the other factor, and all of the resulting partial products are added together, as shown in the margin.

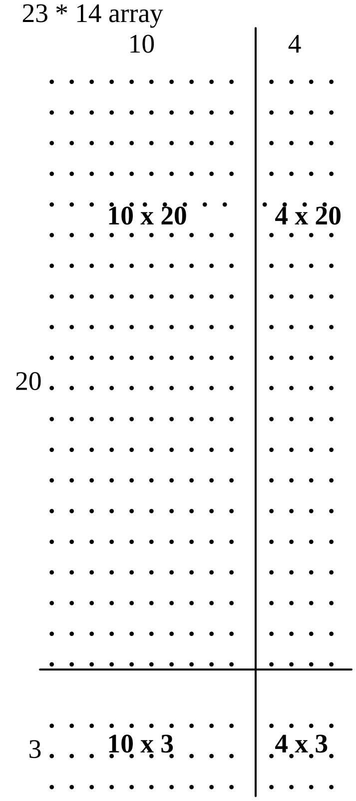
The algorithm can be demonstrated visually with arrays, which are among the first representations of products in *Everyday Mathematics.* The margin shows a 23-by-14 array which represents all of the partial products in 23 *14:

23 *14 = (20 + 3) * (10 + 4)
= (20 * 10) + (20 * 4) + (3 * 10) + (3 * 4)
= 200 + 80 + 30 + 12
= 322

One value of the partial product algorithm is that it previews a procedure for multiplication that is taught in high school algebra*.* Everything is multiplied by everything, and the partial products added. For example:

$(x + 2) * (x + 3) = (x * x) + (2 * x) + (x * 3) + (2 * 3)$
$= x^2 + 5x + 6$

1. Partial Products Algorithm:
23 * 14 array



23 * 14 = (20 + 3) * (10 + 4)
= (20 * 10) + (20 * 4) +
(3 * 10) + (3 * 4)
= 200 + 80 + 30 + 12
= 322

1. Partial Products Algorithm: Algebra-Based Examples

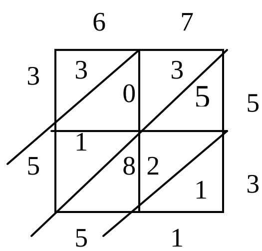
With the rule
$(a + b)^2 = a^2 + 2ab + b^2$
25^2 can be calculated as
$(20 + 5)^2 = 400 + (2*100) + 25 = 625.$

With the rule
$(a + b)(a – b) = a^2 – b^2$,
23 x 17 can be calculated as (20 + 3)(20 – 3) = 400-9= 391

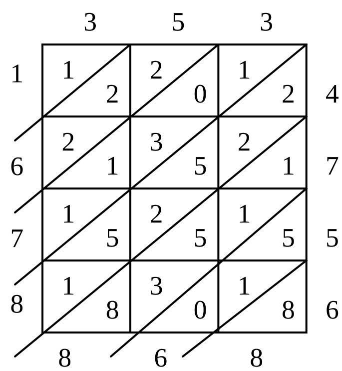
2. Lattice Method

The use of the lattice method has been traced back to India before A.D. 1100. The person using the algorithm writes the partial products within a created lattice and then adds the numerals along each of the diagonals within the lattice. This is a student favorite because of the direct relationship to multiplication facts and its easy expandability to very large numbers.

2. Lattice Method



53 * 67 = 3551



353 * 4756 = 1,678,868

Lattice Method Explanation:

* Draw a lattice with as many boxes horizontally and vertically as digits in the factors of the problem. Write one factor along the top of the lattice and the other along the right side, placing one digit above or beside each box in the lattice.
* Draw diagonals from the upper-right corner of each box, extending beyond the lattice.
* Multiply each digit in one factor by each digit in the other. Write the product in the cell where the corresponding row and column meet. Write the tens digit of the product above the diagonal and the ones digit below the diagonal. For example, since 6 x 5 = 30, write 3 in the box above the diagonal and 0 in the box below the diagonal. If there is not a number in the tens place, a zero can fill the place above the diagonal.
* Start at the bottom right corner and add the digits along each diagonal. Place the sum(s) at the bottom of each diagonal (outside the box) carrying the tens digit to the next diagonal, if needed. The first diagonal contains only 1, so the sum is 1. The sum on the second diagonal is 5 + 2 + 8 = 15. Write only the 5, and carry the one to the next diagonal. The sum along the third diagonal is then 1 + 3 + 0 + 1, or 5. The sum on the fourth diagonal is 3.
* Read the product down the left side and across the bottom, left to right. The product is 3,551.

The second example shows how the lattice method can be easily expanded to accommodate for larger factors.

3. Modified Repeated Addition:

	67
	x 53
	670
	670
	670
50 [67's]	670
or	670
5 [670's]	67
3 [67's]	67
	+ 67
	3551

3. Modified Repeated Addition

Many children are taught to think of whole-number multiplication as repeated addition. However this strategy is inefficient for anything but small numbers. Using a modified repeated addition algorithm, in which multiples of 10, 100, and so on, are grouped together, can simplify the process. For example, it would be tedious to add 67 fifty-three times in order to solve 53 * 67. But if we think of ten 67's as 670, then we can first add the 670's (there are five of them) and then the three 67's.

4. Short Algorithm (Standard) and Modifications

A.	B.
67	67
* 53	* 53
201	201
335	3350
3551	3551

4. Short Algorithm (Standard) and Modified Versions

A. The standard U.S. algorithm.

B. Similar to the standard U.S. algorithm, except the blank is replaced with a zero, which makes it clear that in the second partial product, we are multiplying by 50 (five 10's) and not just by 5.